Impact of Opportunistic Scheduling on Cooperative Dual-Hop Relay Networks

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***Abstract*—This letter advocates the performance of a multiuser relay network (MRN) equipped with a single amplify-and-forward (AaF) relay over Rayleigh fading environments. We derive new expressions for the cumulative distribution function (CDF) of the highest instantaneous end-to-end signal-to-noise ratio (SNR) taking into consideration the two cases of fixed gain relays and variable gain relays. Relying on these statistical results, we derive new expressions for the outage probability and symbol error rate (SER), both of which are obtained in exact closed form. Furthermore, we derive simple asymptotic outage probability and SER. Our asymptotic results confirm that opportunistic scheduling has no impact on the diversity order. We further prove that the array gain is what determines the SNR advantage of opportunistic scheduling over the single user scenario.**

***Index Terms*—Cooperative transmission, fading channels, opportunistic scheduling.**

I. INTRODUCTION

DEPLOYMENT of wireless relays in cooperative transmission has recently appeared as an efficient alternative to extend coverage and combat multipath impairment in wireless networks [1–3]. Among the proposed cooperative strategies [3, 4], amplify-and-forward (AaF) attracts considerable attention due to its ease of implementation and low power consumption. In AaF, the relay simply amplifies the received signal from the source and retransmits a scaled copy of the signal to the destination. As a further categorization, AaF relays can be classified into two subcategories based on the channel state information (CSI) available at the relay: namely variable gain relays and fixed gain relays. Driven by the potential application of such wireless relays, some seminal works have examined the end-to-end performance of *point-to-point dual-hop links* (i.e., single source and single destination with single relay usage) [5, 6].

In point-to-multipoint multiuser applications, for example the case of a cellular system, the base station can select the mobile user with the strongest channel in a time/frequency bin to schedule data transmission. This strategy, which has come to be known as opportunistic scheduling [7–9], can provide a potentially large performance improvement if efficiently utilized. More recently, the concept of cooperative dual-hop transmission has been applied to multiuser wireless downlinks [10, 11]. One example of this is *point-to-multipoint dual-hop links* where a source (or equivalently base station) communicates with many remote and/or geographically scattered destinations (or equivalently mobile users) via a single or multiple relays, as shown in Fig. 1. A few works have been conducted on this architecture [12–15], the results from which have focused only on the capacity and/or throughput performance from the information-theoretic perspective. Despite the demonstrated promised gains of wireless relay networks, the impact of opportunistic scheduling on these networks has not been thoroughly investigated and is not fully understood.

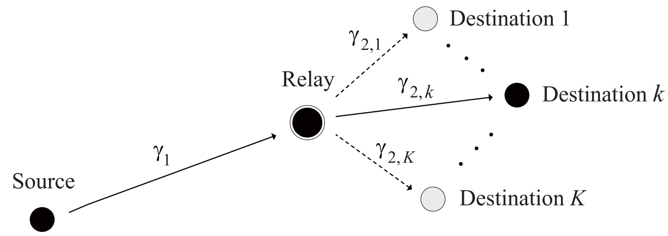

Fig. 1. Illustrative system model for point-to-multipoint dual-hop links.

Motivated by this, we focus our attention on the benefits conferred by opportunistic scheduling in relay-assisted networks. In this letter, we refer to the system architecture depicted in Fig. 1 as a multiuser relay network (MRN). Assuming Rayleigh fading channels, we derive new closed-form expressions for the outage probability and symbol error rate (SER) for fixed and variable gain relays. In doing so, exact expressions are derived for the cumulative distribution function (CDF) of the highest end-to-end SNR link associated with the *strongest* destination. We further derive simple closed-form expressions for the diversity order and array gain. We prove that opportunistic scheduling does not affect the diversity order, rather it increases the array gain and hence reduces the SER. Finally, we demonstrate that increasing the number of destinations shifts the optimal relay location towards the source. In particular, we find that the shift in the optimal relay location is considerable for variable gain relaying, however, less noticeable for fixed gain relaying.

II. PRELIMINARIES AND SYSTEM MODEL

Consider a wireless relay-assisted communication system shown in Fig. 1. A source communicates with K destinations

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with the aid of a single AaF relay. The source has no direct link with the destinations and the transmission is performed only via the relay in a time-division multiple access (TDMA) fashion with two signaling intervals. The source transmits its data signal to the relay in a signaling interval, and in the following signaling interval, the relay retransmits the amplified signal to only one destination which has the most favorable end-to-end channel quality.

Let the modulated signal transmitted by the source denoted as $s(t)$. The received signal at the kth destination is given by

$$y_k(t) = \sqrt{E_{\mathrm{RD}} d_{\mathrm{RD}}^{-\eta}} g_k G \left(\sqrt{E_{\mathrm{SR}} d_{\mathrm{SR}}^{-\eta}} h s(t) + n_r \right) + n_{d_k}, \tag{1}$$

where E_{SR} and E_{RD} denote the average symbol energies at the source and the relay, respectively, and h and g_k represent the channel complex fading coefficients between the source and relay, and between the relay and the kth destination, respectively. The symbols n_r and n_{d_k} represent the additive white Gaussian noise (AWGN) components with one-sided power spectral density N_0 at the relay and the kth destination, respectively. G is defined as the scaling gain applied at the relay. In this letter, the path loss is incorporated in the signal propagation, where d_{SR} is the distance between the source and the relay, d_{RD} is the distance between the relay and the destinations, and η is the path loss exponent. Here we assume that all destinations are equidistant from the relay, and hence d_{RD} is constant for all destinations. The instantaneous end-to-end SNR of the kth destination, $\gamma_{\mathrm{eq},k}$, can be written as [5]

$$\gamma_{\mathrm{eq},k} = \frac{\gamma_1 \gamma_{2,k}}{\gamma_{2,k} + \frac{1}{G^2 N_0}}, \tag{2}$$

where $\gamma_1 = |h|^2 d_{\mathrm{SR}}^{-\eta} E_{\mathrm{SR}}/N_0$ and $\gamma_{2,k} = |g_k|^2 d_{\mathrm{RD}}^{-\eta} E_{\mathrm{RD}}/N_0$ are the per-hop instantaneous SNRs associated with the channels h and g_k. Correspondingly, the per-hop average SNR is given by $\overline{\gamma}_1 = \mathbf{E}[\gamma_1]$ and $\overline{\gamma}_{2,k} = \mathbf{E}[\gamma_{2,k}]$, respectively, where $\mathbf{E}[\cdot]$ is the expectation.

It is obvious from (2) that the choice of the relay gain determines the instantaneous end-to-end SNR. Note that the channel estimation error is inversely proportional to the input SNR [16]. As such, when the source-relay link has a low SNR, a fixed gain constraint of $G^2 = 1/(CN_0)$ is applied at the relay, where $C = \overline{\gamma}_1 + 1$ is a positive constant [6]. This fixed gain relaying alleviates the requirement of full CSI while offering a comparable performance to variable gain relaying. The instantaneous end-to-end SNR of the kth destination employing a fixed gain relay can be rewritten as

$$\gamma_{\mathrm{eq},k,\mathrm{Fix}} = \frac{\gamma_1 \gamma_{2,k}}{\gamma_{2,k} + C}. \tag{3}$$

Another choice of the relay gain is variable gain relaying, which is applied for the case when the source-relay link fading coefficients are precisely estimated at the relay. When the relay does not account for the statistical noise, the variable gain constraint is given by $G^2 = 1/\left(|h|^2 d_{\mathrm{SR}}^{-\eta} E_{\mathrm{SR}}\right)$ [5, 17]. The instantaneous end-to-end SNR of the kth destination employing a variable gain relay can be rewritten as

$$\gamma_{\mathrm{eq},k,\mathrm{Var}} = \frac{\gamma_1 \gamma_{2,k}}{\gamma_1 + \gamma_{2,k}}. \tag{4}$$

We assume that all the destinations are located in a homogeneous environment. In such an environment, the signals from the relay to the K destinations experience independent identically distributed (i.i.d.) Rayleigh fading where all the destinations have the same per-hop average SNR, i.e., $\overline{\gamma}_{2,k} = \overline{\gamma}_2$. We also retain the practical consideration that the dual-hop transmission is subject to independent but not necessarily identically distributed (i.n.d.) Rayleigh fading, i.e., $\overline{\gamma}_1 \neq \overline{\gamma}_2$. As a result, the per-hop instantaneous SNR, $Z = \{\gamma_1, \gamma_{2,k}\}$, follows an exponential distribution, with probability density function (PDF) given by

$$f_Z(\gamma) = \frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma}{\overline{\gamma}_i}}, \tag{5}$$

where $i = 1, 2$. The corresponding CDF of Z can be written as

$$F_Z(\gamma) = 1 - e^{-\frac{\gamma}{\overline{\gamma}_i}}. \tag{6}$$

III. Exact Performance Analysis

Opportunistic scheduling in MRN is achieved by selecting the destination with the highest end-to-end instantaneous SNR out of K destinations, at any particular point in time. The highest instantaneous end-to-end SNR of the selected user (i.e., strongest user), denoted as γ_s, is determined by

$$\gamma_s = \max_{1 \le k \le K} \{\gamma_{\mathrm{eq},k}\}. \tag{7}$$

It is assumed that CSI knowledge of the relay-destination links of the K users are available at the relay. At the relay, channel estimation is conducted based on a pilot sequence sent by the K users. The relay identifies and selects the *strongest* user. The relay then feeds back the index of the *strongest* user to the source.

A common metric for assessing the error performance is the SER. In this letter, we adopt a CDF-based approach and express the SER expression directly in terms of the CDF of γ_s as [18]

$$P_s = \frac{a}{2}\sqrt{\frac{b}{\pi}} \int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} F_{\gamma_s}(\gamma) d\gamma. \tag{8}$$

Our results in this section, apply for all general modulation schemes that have an SER expression of the form $P_s = \mathbf{E}\left[aQ\left(\sqrt{2b\gamma}\right)\right]$, where $\mathbf{E}[\cdot]$ is the statistical average operator. Such modulation schemes include binary PSK (BPSK): $a = 1$ and $b = 1$, and quadrature PSK (QPSK): $a = 1$ and $b = 0.5$.

A. Fixed Gain MRN

The outage probability P_{out} is an important quality of service measure, defined as the probability that γ_s drops below a certain specified SNR threshold γ_{th}. Considering MRN equipped with a fixed gain relay, we can write its outage probability $P_{\mathrm{out,Fix}}$ as

$$P_{\mathrm{out,Fix}} = \Pr\left[\gamma_{s,\mathrm{Fix}} < \gamma_{\mathrm{th}}\right] = F_{\gamma_{s,\mathrm{Fix}}}(\gamma_{\mathrm{th}}), \tag{9}$$

where $F_{\gamma_{s,\mathrm{Fix}}}(\gamma_{\mathrm{th}})$ is the CDF of the highest instantaneous end-to-end SNR of the *strongest* user for fixed gain relaying, evaluated at $\gamma = \gamma_{\mathrm{th}}$. To calculate the outage probability of fixed gain MRN, we first obtain the CDF of $\gamma_{s,\mathrm{Fix}}$.

Theorem 1: Since the CDF of each hop is given by (6), the CDF of $\gamma_{s,\mathrm{Fix}}$ can be expressed as

$$F_{\gamma_{s,\mathrm{Fix}}}(\gamma) =1+2\sqrt{\frac{C\gamma}{\overline{\gamma}_1\overline{\gamma}_2}}e^{-\frac{\gamma}{\overline{\gamma}_1}}\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i} \times\sqrt{K-i}K_1\left(2\sqrt{\frac{(K-i)C\gamma}{\overline{\gamma}_1\overline{\gamma}_2}}\right), \quad (10)$$

where $K_v(x)$ denotes the vth-order modified Bessel function of the second kind.

Proof: See Appendix A. ■

Using (9) and (10), the outage probability of MRN with fixed gain relaying, $P_{\mathrm{out,Fix}}$, is obtained. In the special case of single user dual-hop links, $P_{\mathrm{out,Fix}}$ can be found by setting $K=1$ in (10). This yields the same expression as that in [6, eq. (9)].

Substituting (10) into (8), and using [19, eq. (6.614.5)], the SER of fixed gain MRN is obtained in closed form as

$$P_{s,\mathrm{Fix}} =\frac{a}{2}+\frac{a}{2}\sqrt{\frac{b\overline{\gamma}_1}{1+b\overline{\gamma}_1}}\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}e^{\xi} \times\xi\left(K_1\left(\xi\right)-K_0\left(\xi\right)\right), \quad (11)$$

where $\xi = C(K-i)/2\overline{\gamma}_2\left(1+b\overline{\gamma}_1\right)$. For the single user scenario, the closed-form expression for $P_{s,\mathrm{Fix}}$ is found by setting $K=1$ in (11). Note that this result for single user dual-hop links can also be derived using [6, eq. (10)]. Hence, our result in (11) stands for a generalization of the single user scenario.

By normalizing the total distance between the source and the destinations to unity with $d_{\mathrm{SR}}+d_{\mathrm{RD}}=1$ and representing $\overline{\gamma}_1$ and $\overline{\gamma}_2$ with d_{SR}, we find that $P_{s,\mathrm{Fix}}$ in (11) is a convex function of d_{SR}. As such, the optimal relay location aiming at minimizing the SER can be found by setting the derivative of $P_{s,\mathrm{Fix}}$ with respect to d_{SR} to zero. Although it is intractable to find a closed-form solution for this optimization problem, the optimal relay location can be obtained via a simple line search.

B. Variable Gain MRN

In this subsection, we analyze the performance of variable gain MRN. The outage probability in this case is given by

$$P_{\mathrm{out,Var}} = F_{\gamma_{s,\mathrm{Var}}}\left(\gamma_{\mathrm{th}}\right), \quad (12)$$

where $F_{\gamma_{s,\mathrm{Var}}}(\gamma)$ is the CDF of the highest instantaneous end-to-end SNR of the *strongest* user for variable gain relaying.

Theorem 2: The CDF of each hop is shown in (6), and consequently the CDF of $\gamma_{s,\mathrm{Var}}$ can be expressed as

$$F_{\gamma_{s,\mathrm{Var}}}(\gamma) = 1+\frac{2\gamma}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}\sqrt{K-i} \times e^{-\gamma\left(\frac{1}{\overline{\gamma}_1}+\frac{K-i}{\overline{\gamma}_2}\right)}K_1\left(2\gamma\sqrt{\frac{K-i}{\overline{\gamma}_1\overline{\gamma}_2}}\right). \quad (13)$$

Proof: Following the algebraic steps specified in Appendix A, the final result in (13) is derived. ■

The outage probability can be obtained by substituting (13) into (12). Moreover, $P_{\mathrm{out,Var}}$ for single user scenario can be further simplified by setting $K=1$ in (12). This result is the same as that in [5, eq. (27)].

Substituting (13) into (8) and applying [20, eq. (2.16.1.3)], the closed-form SER expression of variable gain MRN is derived after some manipulations as

$$P_{s,\mathrm{Var}} =\frac{a}{2}+\frac{a}{4}\sqrt{\frac{b\overline{\gamma}_1\overline{\gamma}_2}{2}}\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{5}{2}\right)\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i} \times\tau^{-\frac{1}{2}}{}_2F_1\left(\frac{1}{4},\frac{3}{4};2;1-\frac{4(K-i)\overline{\gamma}_1\overline{\gamma}_2}{\tau^2}\right), \quad (14)$$

where $\tau = \overline{\gamma}_2\left(1+b\overline{\gamma}_1\right)+\overline{\gamma}_1\left(K-i\right)$, $\Gamma(x)$ represents the gamma function, and ${}_2F_1(a,b;c;z)$ denotes the Gauss hypergeometric function. For the single user scenario, $P_{s,\mathrm{Var}}$ is obtained by substituting $K=1$ into (14). We note that this result for single user dual-hop links can also be derived using [5, eq. (13)]. This demonstrates the accuracy and the generality of our result in (14). Similar to fixed gain MRN, we can find the optimal relay location for variable gain MRN.

IV. Asymptotic Performance Analysis

In this section, we derive asymptotic expressions for the outage probability and SER. The new and relatively simple expressions obtained are important to examine the effect of opportunistic scheduling on the diversity order and array gain. It was shown in [21] that the asymptotic SER can be derived using the asymptotic outage probability. As such, we start our analysis by characterizing the asymptotic outage probability, followed by the asymptotic SER.

A. Fixed Gain MRN

To derive the asymptotic outage probability of fixed gain relaying, we first note that $C=1+\overline{\gamma}_1\rightarrow\overline{\gamma}_1$ as $\overline{\gamma}_1,\overline{\gamma}_2\rightarrow\infty$. As such, at high SNRs, by setting $\overline{\gamma}_2=\kappa\overline{\gamma}_1$ and $\overline{\gamma}_1=\gamma_{\mathrm{th}}/\lambda$ in (9), $P_{\mathrm{out,Fix}}$ can be rewritten as

$$P_{\mathrm{out,Fix}} =P(\lambda) =1+e^{-\lambda}\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}\omega K_1\left(\omega\right), \quad (15)$$

where $\omega=2\sqrt{((K-i)\lambda/\kappa)}$ and $\lambda,\ \kappa\in\mathbb{R}^{+}$. By comparing (9) and (15) we find that the behavior of $P_{\mathrm{out,Fix}}$ for large $\overline{\gamma}_1$ and $\overline{\gamma}_2$ is equivalent to the behavior of $P(\lambda)$ around $\lambda=0$. To proceed with our analysis we express the exponential and Bessel functions in (15) in terms of the Taylor series expansion around $\lambda=0$. After further algebraic calculations, we obtain

$$P(\lambda)=A(\lambda)+B(\lambda), \quad (16)$$

where

$$A(\lambda)=1+\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}\sum_{p=0}^{\infty}\frac{(-\lambda)^p}{p!}, \quad (17)$$

and

$$B(\lambda)=\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-1)^p\left(\frac{K-i}{\kappa}\right)^{q+1}}{p!q!(q+1)!} \times\lambda^{p+q+1}\left[\ln\left(\frac{(K-i)\lambda}{\kappa}\right)-\psi(q+1)-\psi(q+2)\right], \quad (18)$$

where $\psi(x)$ is the psi function defined in [19, eq. (8.36.1)].

We next find the first nonzero derivative order of $A(\lambda)$ and $B(\lambda)$ and discard the higher order terms. After performing some algebraic manipulations, we have

$$A(\lambda) = \lambda + o(\lambda), \tag{19}$$

and

$$B(\lambda) = \frac{\lambda}{\kappa}\Xi[K]_1 + o(\lambda), \tag{20}$$

where $\Xi[K]_1 = \sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}(K-i)\ln((K-i)\lambda/\kappa)$.

By substituting $\lambda = \gamma_{\text{th}}/\overline{\gamma}_1$ into (19) and (20), the asymptotic outage probability is derived as

$$P_{\text{out,Fix}}^{\infty} = \frac{\gamma_{\text{th}}}{\overline{\gamma}_1}\left(1 + \frac{1}{\kappa}\Xi[K]_2\right) + o\left(\overline{\gamma}_1^{-1}\right), \tag{21}$$

where $\Xi[K]_2 = \sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}(K-i)\ln((K-i)\gamma_{\text{th}}/\overline{\gamma}_2)$ and $\kappa = \overline{\gamma}_2/\overline{\gamma}_1$.

Using (21), we perform some basic algebraic manipulations to obtain the asymptotic SER expression given by

$$P_{s,\text{Fix}}^{\infty} = (G_{a,\text{Fix}}\overline{\gamma}_1)^{-G_{d,\text{Fix}}} + o\left(\overline{\gamma}_1^{-G_{d,\text{Fix}}}\right), \tag{22}$$

where the diversity order is $G_{d,\text{Fix}} = 1$ and the array gain is

$$G_{a,\text{Fix}} = \left(\frac{a}{4b} + \frac{a}{4b\kappa}\Xi[K]_3\right)^{-1}, \tag{23}$$

where $\Xi[K]_3 = \sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}(K-i)\ln((K-i)/\overline{\gamma}_2)$. It is evident from (22) that for fixed gain relaying, the diversity order is not influenced by opportunistic scheduling. However, noting that $\Xi[K]_3 - \Xi[K+1]_3 > 0$ for any K, we verify that $\Xi[K]_3$ is a monotonically decreasing function of K. As such, the effect of opportunistic scheduling is to improve the array gain, thereby reducing the SER. In Section V, we demonstrate that an SNR improvement of at least 10 dB is achieved with opportunistic scheduling.

B. Variable Gain MRN

To derive the asymptotic outage probability of variable gain relaying, we first substitute $\overline{\gamma}_2 = \kappa\overline{\gamma}_1$ and $\overline{\gamma}_1 = \gamma_{\text{th}}/\lambda$ into (12) to yield

$$\begin{aligned} P_{\text{out,Var}} =& Q(\lambda) \\ =& 1 + \sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}e^{-\lambda-\frac{\chi}{4\lambda}}\chi K_1(\chi), \end{aligned} \tag{24}$$

where $\chi = 2\lambda\sqrt{(K-i/\kappa)}$. Using the Taylor series expansion of the exponential and Bessel functions, and following the same algebraic steps as described in Section IV-A, we derive the asymptotic outage probability as

$$P_{\text{out,Var}}^{\infty} = \begin{cases} \frac{\gamma_{\text{th}}}{\overline{\gamma}_1}\left(1 + \frac{1}{\kappa}\right) + o\left(\overline{\gamma}_1^{-1}\right), & K = 1, \\ \frac{\gamma_{\text{th}}}{\overline{\gamma}_1} + o\left(\overline{\gamma}_1^{-1}\right), & K \geq 2, \end{cases} \tag{25}$$

where $\kappa = \overline{\gamma}_2/\overline{\gamma}_1$. By inserting (25) into (8), we derive the asymptotic SER expression as

$$P_{s,\text{Var}}^{\infty} = (G_{a,\text{Var}}\overline{\gamma}_1)^{-G_{d,\text{Var}}} + o\left(\overline{\gamma}_1^{-G_{d,\text{Var}}}\right), \tag{26}$$

where the diversity order is $G_{d,\text{Var}} = 1$ and the array gain is

$$G_{a,\text{Var}} = \begin{cases} \frac{4b\kappa}{a(1+\kappa)}, & K = 1, \\ \frac{4b}{a}, & K \geq 2. \end{cases} \tag{27}$$

The result in (26) confirms that for variable gain relaying, opportunistic scheduling has no impact on the diversity order. However, by inspecting (27), we see that the effect of opportunistic scheduling is to increase the array gain. This array gain is as high as 18 dB, as we will show in Section V.

C. Fixed versus Variable Gain Relaying

We now compare the asymptotic SER of fixed gain relaying with that of variable gain relaying. We first remark that both fixed gain relaying and variable gain relaying achieve the same diversity order. The main fundamental difference between the two relaying protocols lies in the array gain, which is indicated by (23) and (27). To characterize this difference, we present the ratio of their array gains as

$$\frac{G_{a,\text{Fix}}}{G_{a,\text{Var}}} = \begin{cases} \frac{\overline{\gamma}_1+\overline{\gamma}_2}{\overline{\gamma}_2+\overline{\gamma}_1\ln\overline{\gamma}_2}, & K = 1 \\ \frac{\overline{\gamma}_2}{\overline{\gamma}_2+\overline{\gamma}_1\Xi[K]_3}, & K \geq 2. \end{cases} \tag{28}$$

By observing (28), we find that when $K = 1$, the performance gap between fixed and variable gain relaying is determined by the values of $\overline{\gamma}_1$ and $\overline{\gamma}_2$. When $K \geq 2$, the ratio tends towards one for large K. This reveals that the performance of fixed gain relaying approaches that of variable gain relaying in the large K limit.

V. Numerical Results

Numerical examples and simulations are carried out to demonstrate the accuracy of our proposed analysis and the effectiveness of opportunistic scheduling in Rayleigh fading conditions. Throughout this section, we normalize the total distance between the source and the destinations to unity such that $d_{\text{SR}} + d_{\text{RD}} = 1$. The variance of the fading coefficients is also normalized to unity with $\mathbf{E}\left[|h|^2\right] = 1$ and $\mathbf{E}\left[|g_k|^2\right] = 1$. Moreover, equal average energies are assumed at the source and the relay, i.e., $E_{\text{SR}} = E_{\text{RD}}$. Therefore, the first hop and the second hop average SNR attenuates by $d_{\text{SR}}^{-\eta}$ and $(1-d_{\text{SR}})^{-\eta}$, respectively. In this section our results concentrate on $\eta = 4$. In addition, BPSK modulation is considered in all the figures.

Fig. 2 presents the outage probability for both fixed and variable gain relaying as a function of γ_{th}. The curves are plotted for $K = 1$ and $K = 5$. We find that the points generated via Monte Carlo simulations match precisely with the analytic curves, highlighting the accuracy of our analysis. As expected, selecting the highest end-to-end SNR link in a multiuser scenario (i.e., $K = 5$) yields superior performance compared to that of a single user scenario (i.e., $K = 1$).

Fig. 3 presents the exact SER against d_{SR} to investigate the impact of the relay placement on the error performance. For the single user scenario with $K = 1$, we observe that the optimal relay location for variable gain relaying is halfway between the source and the destinations at $d_{\text{SR}} = 0.5$. However, the optimal relay location for fixed gain relaying is at $d_{\text{SR}} = 0.6$. This can be explained by the fact that the instantaneous end-to-end SNR of fixed gain relaying given by

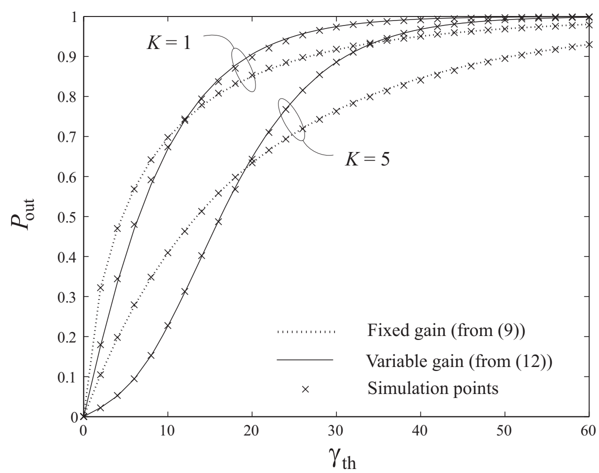

Fig. 2. Outage probability of MRN for fixed and variable gain relays: $E_{\mathrm{SR}}/N_0 = 0$ dB, $d_{\mathrm{SR}} = 0.4$, and $d_{\mathrm{RD}} = 0.6$.

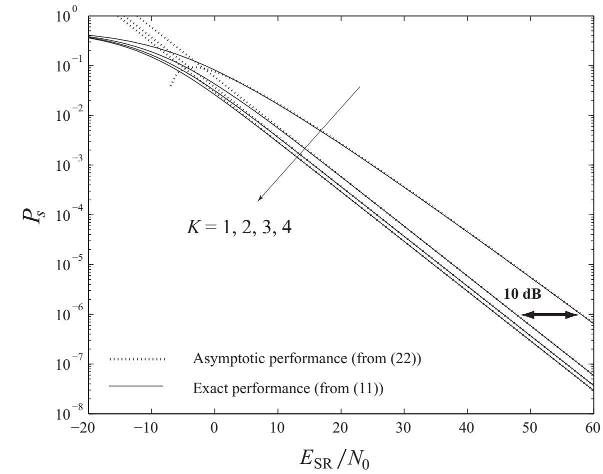

Fig. 4. Exact and asymptotic SER of MRN for fixed gain relays: $d_{\mathrm{SR}} = 0.3$ and $d_{\mathrm{RD}} = 0.7$.

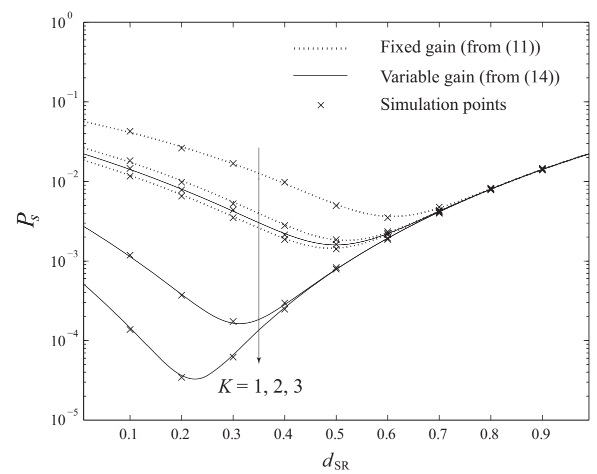

Fig. 3. Exact SER of MRN for fixed and variable gain relays: $E_{\mathrm{SR}}/N_0 = 10$ dB.

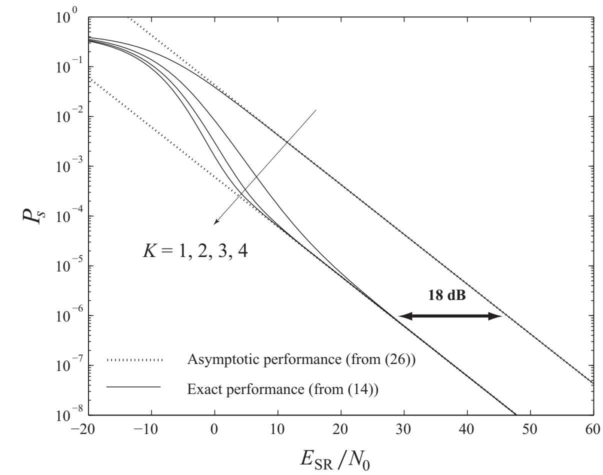

Fig. 5. Exact and asymptotic SER of MRN for variable gain relays: $d_{\mathrm{SR}} = 0.3$ and $d_{\mathrm{RD}} = 0.7$.

(3) is asymmetric caused by the constant C in the denominator. For the multiuser scenario, we observe that the optimal relay location is shifted towards the source. For example when $K = 3$, the optimal relay location is shifted towards the source at $d_{\mathrm{SR}} = 0.22$ for variable gain relaying, and $d_{\mathrm{SR}} = 0.49$ for fixed gain relaying. It is evident that with the increasing number of users, the shift in the optimal relay location is more salient in variable gain relaying than fixed gain relaying.

Figs. 4 and 5 present the SER of fixed gain and variable gain relaying, respectively, versus E_{SR}/N_0 for various K. The exact SER and the asymptotic SER are compared. As expected, the asymptotic expressions well approximate the exact expressions in the high SNR regime. As predicted from the asymptotic expressions in (22) for fixed gain relaying and (26) for variable gain relaying, we see that the diversity order is unaffected by opportunistic scheduling. However, we confirm that opportunistic scheduling has an obvious SNR advantage over the single user scenario for both fixed and variable gain relaying, which is explicitly indicated by the array gain expressions in (23) and (27). At the SER of 10^{-6}, Fig. 4 shows that for fixed gain relaying, $K = 2$ outperforms the single user scenario by 10 dB. Further increasing K brings marginal benefits to the array gain. On the other hand, Fig. 5 shows that for variable gain relaying, $K = 2$ is superior by 18 dB to the single user scenario. However, further increasing K leads to array gain saturation.

VI. Conclusion

The performance gains offered by opportunistic scheduling in a multiuser relay network have been examined for both fixed and variable gain relays. Capitalizing on our new exact closed-form expressions for the CDF of the *strongest* end-to-end SNR link in a multiuser scenario, exact expressions for the SER have been derived in closed form by following a unified CDF-based approach. It has been shown that our analysis can be viewed as a generalization of the single user dual-hop link. Furthermore, by utilizing the behavior of the CDF, asymptotic SER expressions have been derived in the high SNR regime to explicitly reveal the impact of opportunistic scheduling on the SER. We have proved that opportunistic scheduling has no effect on the diversity order but leads to a noticeable increase in the array gain.

Appendix A
Proof of Theorem 1

We now calculate the CDF of $\gamma_{s,\mathrm{Fix}}$. Assuming that the K relay-destination links undergo i.i.d. Rayleigh fading, the CDF of $\gamma_{s,\mathrm{Fix}}$ is given by

$$\begin{aligned}F_{\gamma_{s,\mathrm{Fix}}}(\gamma) &= \int_0^\infty \Pr\left[\gamma_{\mathrm{eq},k,\mathrm{Fix}} < \gamma|\gamma_1\right]^K f_{\gamma_1}(\gamma_1)d\gamma_1 \\ &= \int_0^\infty \Pr\left[\frac{\gamma_1\gamma_{2,k}}{\gamma_{2,k}+C} < \gamma|\gamma_1\right]^K f_{\gamma_1}(\gamma_1)d\gamma_1 \\ &= \int_0^\gamma \Pr\left[\gamma_{2,k} > \frac{C\gamma}{\gamma_1-\gamma}|\gamma_1\right]^K f_{\gamma_1}(\gamma_1)d\gamma_1 \\ &\quad + \int_\gamma^\infty \Pr\left[\gamma_{2,k} < \frac{C\gamma}{\gamma_1-\gamma}|\gamma_1\right]^K f_{\gamma_1}(\gamma_1)d\gamma_1 \\ &= I_1 + I_2, \end{aligned} \tag{29}$$

where

$$I_1 = \int_0^\gamma \frac{1}{\overline{\gamma}_1} e^{-\frac{\gamma_1}{\overline{\gamma}_1}} d\gamma_1 = 1 - e^{-\frac{\gamma}{\overline{\gamma}_1}}, \tag{30}$$

and

$$I_2 = \int_\gamma^\infty \left[1 - e^{-\frac{C\gamma}{\overline{\gamma}_2(\gamma_1-\gamma)}}\right]^K \frac{1}{\overline{\gamma}_1} e^{-\frac{\gamma_1}{\overline{\gamma}_1}} d\gamma_1. \tag{31}$$

Using the binomial expansion in [19, eq. (1.111)], I_2 can be evaluated as

$$\begin{aligned}I_2 &= \frac{1}{\overline{\gamma}_1}\sum_{i=0}^{K}\binom{K}{i}(-1)^{K-i}\int_\gamma^\infty e^{-\frac{(K-i)C\gamma}{\overline{\gamma}_2(\gamma_1-\gamma)}} e^{-\frac{\gamma_1}{\overline{\gamma}_1}} d\gamma_1 \\ &= e^{-\frac{\gamma}{\overline{\gamma}_1}} + \frac{1}{\overline{\gamma}_1}\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i}\int_\gamma^\infty e^{-\frac{(K-i)C\gamma}{\overline{\gamma}_2(\gamma_1-\gamma)}} e^{-\frac{\gamma_1}{\overline{\gamma}_1}} d\gamma_1 \\ &= e^{-\frac{\gamma}{\overline{\gamma}_1}} + \frac{1}{\overline{\gamma}_1}\sum_{i=0}^{K-1}\binom{K}{i}(-1)^{K-i} I_3. \end{aligned} \tag{32}$$

By setting $\lambda = \gamma_1 - \gamma$, we reexpress the integral I_3 as

$$I_3 = e^{-\frac{\gamma}{\overline{\gamma}_1}} \int_0^\infty e^{-\frac{(K-i)C\gamma}{\overline{\gamma}_2\lambda}} e^{-\frac{\lambda}{\overline{\gamma}_1}} d\lambda. \tag{33}$$

Applying [19, eq. (3.324.1)] to (33), the integral I_3 can be solved as

$$I_3 = 2e^{-\frac{\gamma}{\overline{\gamma}_1}} \sqrt{\frac{(K-i)C\overline{\gamma}_1\gamma}{\overline{\gamma}_2}} K_1\left(2\sqrt{\frac{(K-i)C\gamma}{\overline{\gamma}_1\overline{\gamma}_2}}\right), \tag{34}$$

and thus I_2 is obtained by inserting (34) into (32).

Finally, substituting (30) together with (32) into (29) leads to $F_{\gamma_{s,\mathrm{Fix}}}(\gamma)$ presented in (10) which is the desired result.

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